The Effectiveness of Multidimensional Scaling in TSPs Whose Metric is not Euclidean

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Introduction

- There is strong evidence from human performance on **traveling salesperson problems (TSPs)** that the human visual system represents 2D images as 2D Euclidean planes. (Fig. 1a)
- Model-fitting suggests that human subjects produce TSP tours in a sequence of coarse-to-fine approximations by using a **hierarchical clustering (pyramid)** representation of the problem.
- When obstacles are introduced into a 2D Euclidean TSP, the distances between cities are no longer Euclidean, but human subjects can still produce near-optimal tours. (Fig. 1b)
- **Multidimensional scaling (MDS)** is a classical method for recovering a Euclidean approximation of a space given a set of pairwise dissimilarities between objects. It is commonly assumed that MDS is a powerful tool for modeling cognitive spaces.
- Can human performance on the **traveling salesperson problem with obstacles (TSP-O)** be approximated by applying a pyramid to the Euclidean approximation of the problem produced by MDS? (Fig. 1c)
- Can this model be extended to work on problems with **complex obstacles** or **non-metric distances**?

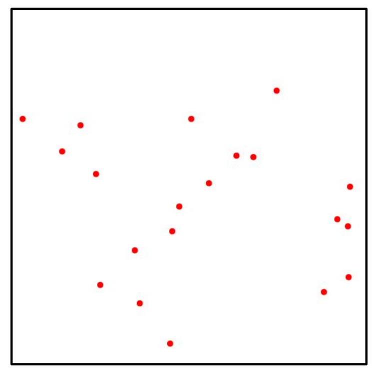

Fig. 1a. TSP

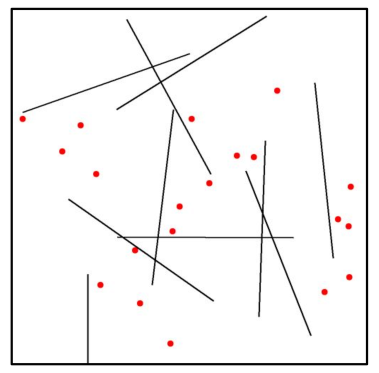

Fig. 1b. With Obstacles

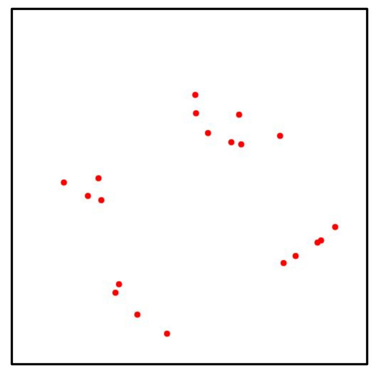

Fig. 1c. MDS Reconstruction

Preliminary Simulations

- The TSPs used in the experiments were generated from uniform-randomly distributed cities on a **500x500 pixel space**, with **10 obstacles** introduced at uniform-randomly distributed positions and orientations. (See Fig. 1b)
- The number of cities and lengths of obstacles varied: from **10-50 cities** in increments of 10 and **100-300 pixel obstacles** in increments of 50. **30 problems** were generated for each set of parameters.
- We applied **Concorde** (optimal - top panels below) and **pyramid** (approximate - bottom panels) TSP solvers in conjunction with **{2-10}-dimensional MDS**. Performance did not improve beyond **4D** space. The graphs below show results for 2D and 3D MDS approximations.
- The performance of the two solvers when they were applied to MDS approximations is represented as a percentage error relative to the optimal tour for the TSP-O.
- Results showed that **250 pixel obstacle problems** were most discriminative between conditions. **Experiments 1 and 2** used a subset of these problems.

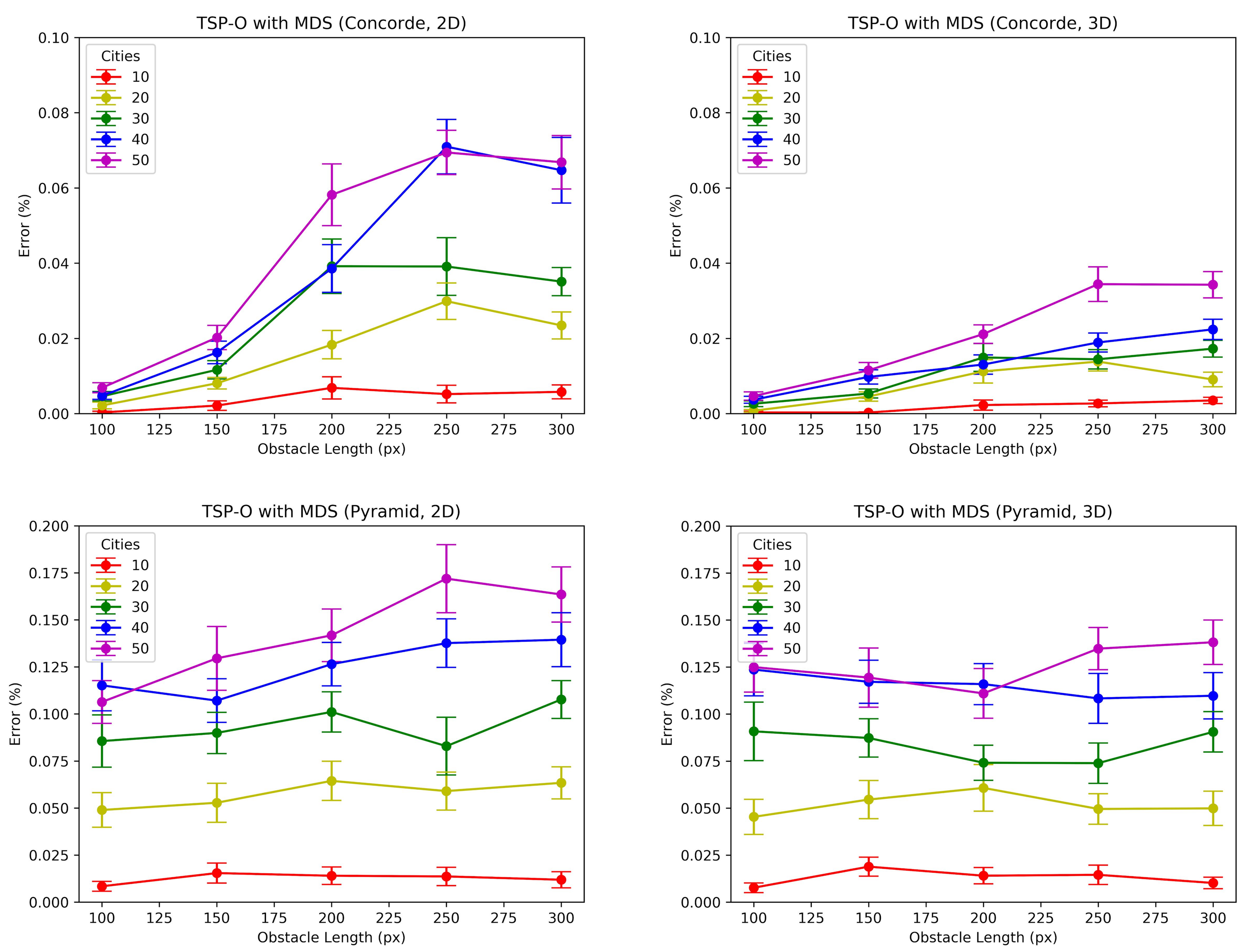

Fig. 2

Experiment 1: Human Performance on TSP-O

- The subject was tested on the **TSP-Os** with 250 pixel obstacles and 20 and 50 cities (30 problems each). (Fig. 3a shows one such 20-city problem.)
- The subject was also tested on the Euclidean TSPs produced by applying **2D MDS** to the distance matrices of the TSP-Os. (Fig. 3b shows the same 20-city problem. Note the clustering produced by MDS to represent the presence of obstacles.)
- Fig. 3c shows how the 2D MDS tour in 3b would look if applied to the actual TSP-O. The error for MDS tours is calculated from the tour in 3c.
- Fig. 4 is a scatterplot representing the relation of distances as reconstructed by 2D MDS with the true distances with obstacles (for one example TSP-O).
- **Results:** The subject's performance on TSP-O is different from his performance on 2D MDS (Fig. 5). More subjects are needed in order to substantiate this observation.

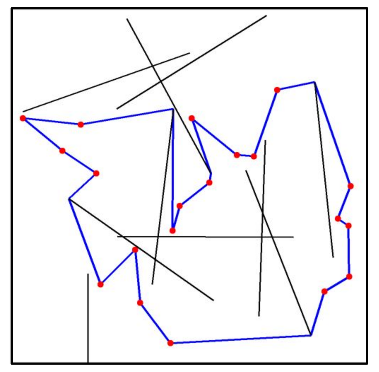

Fig. 3a. TSP-O Tour

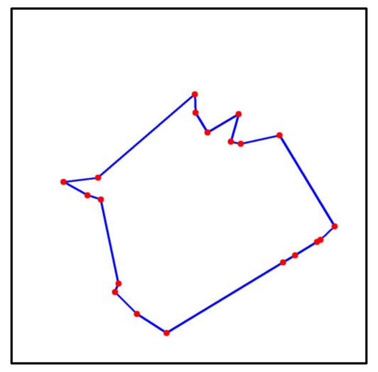

Fig. 3b. 2D MDS Tour

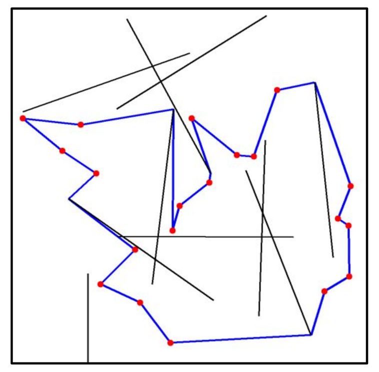

Fig. 3c. Tour from 3b shown with obstacles

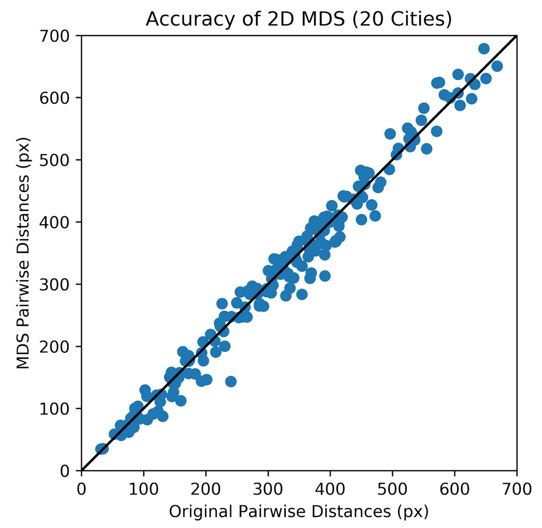

Fig. 4

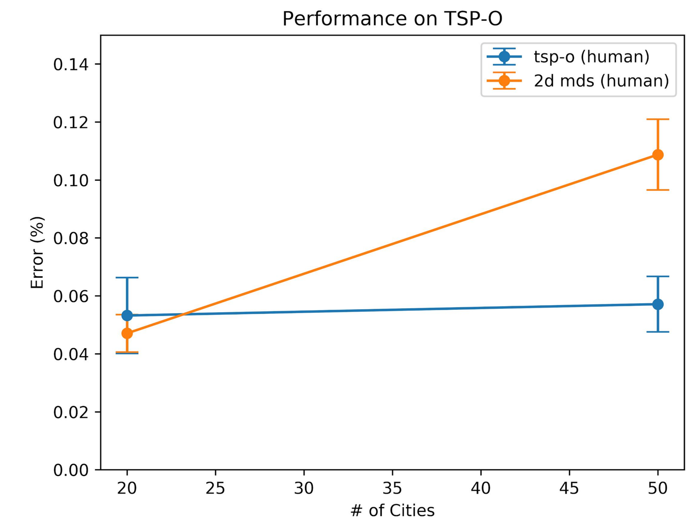

Fig. 5

Experiment 2: Concorde and Pyramid with 2D, 3D, 4D MDS

- Hypothetically, humans can use cognitive spaces of dimensions higher than 2 or 3. But subjects cannot be tested in any meaningful way with visual stimuli having dimensions higher than 3.
- Both Concorde and pyramid can be applied to a space with an arbitrary number of dimensions.[4] In order to gain insight into the cognitive mechanisms that humans use, one can compare the performance of these models applied to MDS in higher dimensions with the performance of subjects on the TSP-Os.
- **Results:** As already shown in Fig. 2, both Concorde's and pyramid's performance deteriorates with a higher number of cities (Fig. 6). Human performance does not seem to show the same effect.
- Both models' performances improve when applied to higher dimensional spaces.
- It seems that the MDS representation is not an adequate model of human cognitive representation for TSP-O.

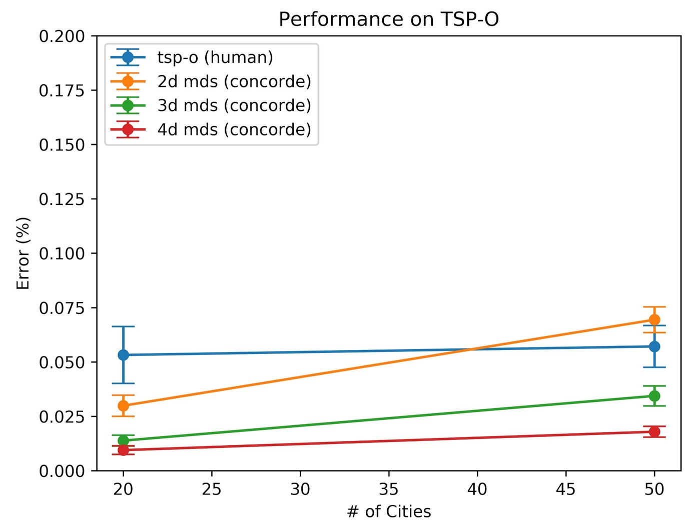

Fig. 6a

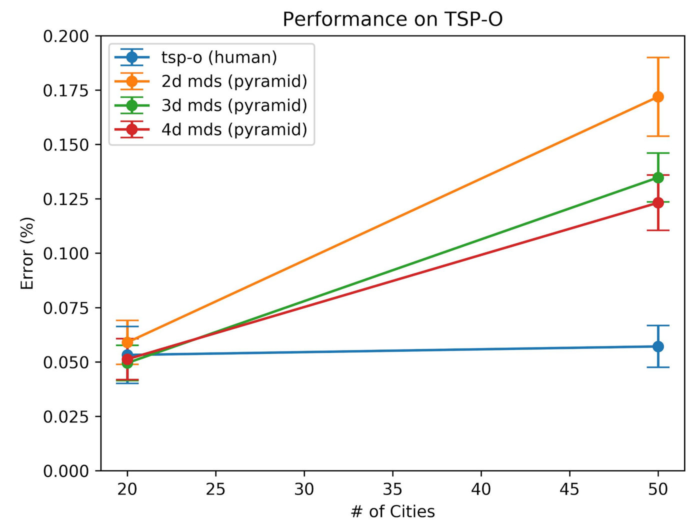

Fig. 6b

Experiment 3: Complex Obstacles and Non-Metric TSPs

- As shown in [3], humans subjects can effectively solve non-metric TSPs. Following this, we tested our models with one particular type of non-metric TSP.
- This non-metric TSP was constructed by assigning one color to the cities in the center of the image and another color to the cities outside of the center. The border between the two regions was a square. The distance between cities of different colors was twice the Euclidean distance. As a result, the triangle inequality was violated for a number of triangles. (Fig. 7b)
- For comparison, we constructed metric TSP-Os with the same cities and one square obstacle separating the regions. The square had two small openings on opposite sides. (Fig. 7a)
- We applied MDS to both types of problems, and used Concorde and pyramid to solve them.
- **Results:** MDS is much more effective in 2D representation when distances are metric, but this difference tends to disappear in higher dimensions.

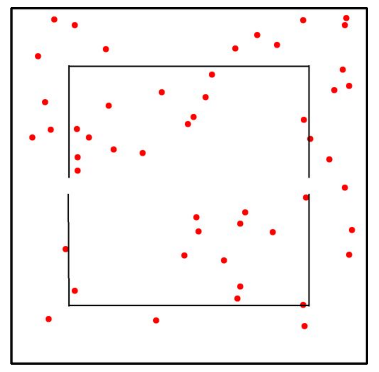

Fig. 7a. Complex TSP-O

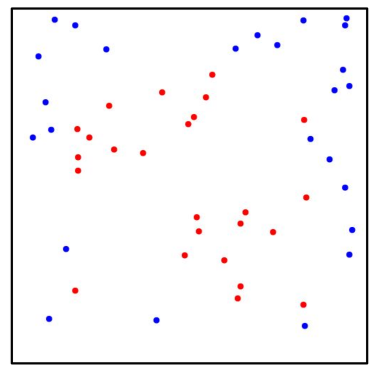

Fig. 7b. Two-Color TSP

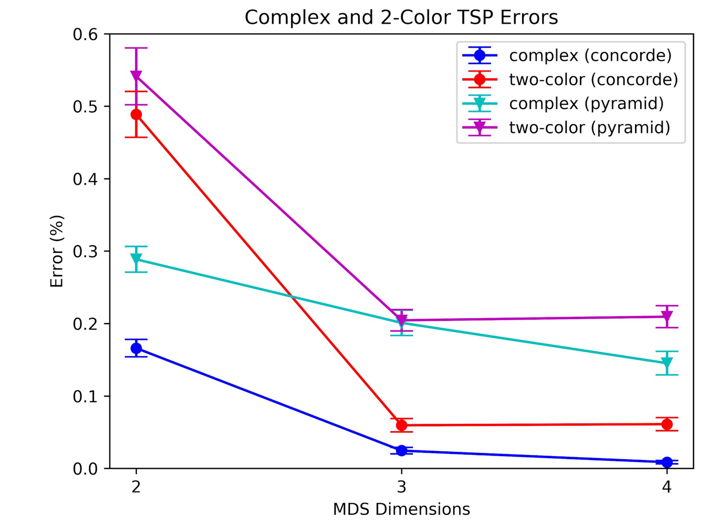

Fig. 8

Future Work

- **Can the pyramid algorithm be improved?** The model used in this experiment is a graph-pyramid with clusters produced based on the minimum spanning tree of the problem graph. A visual-pyramid, with clusters produced based on the varying intensity of the distribution of cities, could yield equal or better performance.[5]
- **Can a better scaling algorithm be employed?** Our provisional answer to this is **no**. The experiments have shown that MDS is very good at reconstructing the pairwise distances, even in low-dimensions.
- **Could humans be using an algorithm different from pyramid?** The performance of Concorde on MDS provides a lower bound which human performance falls above. So, it is possible that humans are using (higher-dimensional) MDS representations, but with an altogether different solving algorithm.
- **Could humans be using a different representation?** One possibility is that humans are applying a **local MDS** algorithm, in which only the areas of the problem which are directly affected by obstacles are reconstructed. This would share the intuition behind previous TSP-O models and conjectures about pyramid solvers.[6,7]

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Notes

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[2] Department of Cognitive Sciences; University of California, Irvine.

[3] See Sajedinia, Pizlo, and Hélie at CogSci next week.

[4] Haxhimusa et al. 2011.

[5] Pizlo et al. 2006.

[6] Saalweachter and Pizlo 2008.

[7] Pizlo and Stefanov 2013.